Basic definitions and concepts in Mathematics

1. **Set**

   Set is a basic concept in Mathematics; it is a collection of well-defined objects.

   S={x|P(x)}: set S contains elements *x's* which has the property *P(x)*

   E.g.: set of odd numbers O={1,3,5…} or O={n|n=2·m+1 where m=1,2,3,4…}, people in a city taller than 185 cm

2. **Union of two sets:**
   Union of set A and B is a set denoted with A∪B, which contains elements (x) belong to either A or B or both.
   A∪B={x| x∈A or x∈B} (x∈A: x is in set A; x∉B: is not in set B)
   A={1,3,5,7,9}, B={3,6,9}
   A∪B={1,3,5,6,7,9}

3. **Intersection of two sets**
   Intersection of set A and B is a set denoted with A∩B, which contains elements (x) belong to both A and B.
   A∩B={x| x∈A and x∈B}
   A={1,3,5,7,9}, B={3,6,9}
   A∩B={3,9}

4. **Subset**
   Set S is subset of set U if, and only if, every element of S is an element of T. (All elements of S is the element of T but T can have elements, which are not in S)
   S⊆U or U⊇S
   U={1,3,5,7,9}, S={3,9}⇒ S⊆U

5. **Empty or null set (∅), universal set (U)**
   Empty set has no elements, so it is the subset of all sets.
   Universal set is the totality of all elements under consideration.

6. **Set of Numbers**
   $\mathbb{N}: natural\ numbers: 1,2,3\ \ldots$
   $\mathbb{Z}: integers:\ \ldots - 2, -1, 0, 1, 2\ \ldots$
   $\mathbb{Q}: rational\ numbers: \ldots - \frac{11}{9}, 0, \frac{3}{7}, 1 = \frac{7}{7}, \ldots$
   $\mathbb{I}: irrational\ numbers: \sqrt{3},\ \pi, e\ (Euler's\ number), -\sqrt[5]{7}$, the not rational numbers
   $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}: real\ numbers: every\ numbers\ above$

7. **Definition of a function**
   Function *f* from a set A into a set B ($f: A \to B$) is a correspondence that assigns each element *x* from A (x∈A) exactly one element *y* from B (*y*∈B): *y* is called the image of *x* under *f* and denoted with *f(x)*. Domain of *f (notation: D_f)* is A, and the range *f(A)={f(x) where x∈A} (R_f)*.

8. A **linear equation in one variable** *x* is an equation that can be written in the standard form

$ax + b = 0$ where a and b are real numbers with $a \neq 0$.

The solution is: $x = -\frac{b}{a}$

9. A **quadratic equation** in is an equation that can be written in the general form
$$ax^2 + bx + c = 0 = (x - x_1)(x - x_2)$$

where a, b and c are real numbers, with $a \neq 0$. A quadratic equation is also known as a **second-degree polynomial equation**.

Solution is
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

10. **<u>Definition and identities of power</u>**
The n^{th} *power of x:* $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ where total number of *x* is *n, n is a natural number.*

1. $x^n \cdot x^m = x^{(n+m)}$
2. $x^n \cdot y^n = (x \cdot y)^n$
3. $(x^n)^m = x^{n \cdot m}$
4. $\frac{1}{x^n} = x^{-n}$
5. $x^0 = 1,\ x^1 = x$

11. **<u>Definition and identities of radical</u>**
The n^{th} *root of x:* $\sqrt[n]{x} = y\ if\ y^n = x$ where $x \geq 0, n \geq 1$

1. $x^{\frac{1}{n}} = \sqrt[n]{x}$
2. $\sqrt[n]{x} \cdot \sqrt[m]{x} = \left(\sqrt[n \cdot m]{x}\right)^{n+m}$
3. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[n \cdot m]{x}$
4. $\sqrt[n]{x}\sqrt[n]{y} = \sqrt[n]{x \cdot y}$

12. **<u>Definition and identities of logarithm</u>**
Logarithm x to base a equals y, shortly: $\log_a x = y\ if\ y^a = x$ where $a \neq 1$, $a>0$, $b > 0$.

1. $\log_a x + \log_a y = \log_a (x \cdot y)$
2. $\log_a x^c = c \cdot \log_a x$
3. $\log_a x = \frac{\log_c x}{\log_c a}$
4. $\log_a x - \log_a y = \log_a \frac{x}{y}$
5. $\log_a 1 = 0;\ \log_a a = 1$

13. **Definition and identities of exponential**

$f(x) = a^x$ function is called exponential function, where '*a*' *is constant and a real number, x, y are real number*

1. $a^x \cdot a^y = a^{(x+y)}$
2. $a^x \cdot b^x = (a \cdot b)^x$
3. $(a^x)^y = a^{x \cdot y}$
4. $\frac{1}{a^x} = a^{-x}$